LEARNING THE CHAIN FOUNTAIN IN STEADY STATE MOTION

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ABSTRACT

A chain of metallic balls or spheres leaps over and above the rim of a container in which it rests tracing a path like a fountain when one end of the chain is given a push towards the ground. This interesting phenomenon was discovered by Steve Mould to describe at atomic level what happens to the molecules in a self-siphoning fluid. However, to his surprise, the chain of beads did not flow over the rim of the container as the self-siphoning fluid did.

But why does this chain do this and what forces cause this fountain? Propositions for the origin of the force that cause of this fountain motion have been presented by Dr. Biggins and Dr. Warner as well as Rogério Martins. Analysis and study of the motion of the chain as it traces this fountain path will help towards the understanding of the origin and nature of this force that leads to the leap into a fountain.

The experiment shall be carried out, and the motion filmed to assist in the study and analysis of this rather interesting effect and the data collected and plots made. Different arrangements of the chain will be attempted, and computer simulation of the motion will assist in the understanding and learning of this entertaining Mould Effect.

It is my objective that at the end of the process – from reviewing previously published literature, undertaking of the experiment with different data collection methods, calculation and simulation – a good understanding of the chain fountain shall be better grasped and appreciated.

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CHAPTER I: INTRODUCTION

Chains are one of the most widely used technologies. They have been used in several ways for several applications such as power transfer or traction, or even for decoration. Chains are typically used when a slack pile of chain is stretched out in such a way that it is then subjected to tension, rather than compression. Chains are employed in functions much like the regular rope, as they also have great strength in tension but very little, if any in compression. However, Mould, in a rather awestriking video, demonstrated that when one end of a long chain held in an elevated container is pulled down to the ground, the chain will not just flow out of the container and down to the ground, but will spontaneously leap out of the container, over and above the rim forming a fountain. As entertaining and easy as this may seem to be, it poses particularly the question as to what the origin of the force that causes this spontaneous leap over and above the rim of the container in which the chain rests; why does the chain leap into a fountain? Several related chain problems have served as beginnings in the attempts to guide the understanding of this Mould effect, and these include problems related to falling chains (top pile and bottom pile), as well as U-chains or folded chains.

To understand a bit about chains, we employ the general tools of dynamics to the motion of chains as discussed here. A force applied to a body by pulling for example on a rope tied to that body creates momentum and kinetic energy, which are both quantities that a body possesses due to its velocity. Considering a force applied on a chain to pull it into motion, the chain having a length s with a mass per unit length λ; the mass of the chain is λs. If the speed of the chain is v, we shall have that the momentum is λsv. Likewise, the kinetic energy of the chain is $\frac{1}{2}\lambda s v^2$. Given that the chain is being pulled from an initial position in a stationary pile with a speed v, then the length s of the chain already out of the pile increases at a rate $ds/dt = v$. This means that the momentum changes at a rate of $\frac{d(\lambda s v)}{dt} = \lambda v \frac{ds}{dt} = \lambda v^2$. This is the force required to accelerate the chain to a speed v.

Now let us consider the motion of chains and ropes around corners. First, we recall the nature of the tension force in a rope or chain. When a pulling force is applied to a chain or rope, internal forces transmit this force in along the length of the chain such that at any point within the length of the chain there is a resultant force of zero. This leaves an equal but opposite force to be transmitted to the other end of the rope. It is this nature of the tension force that allows for the extensive use of ropes and chains in tension or pulling on something rather than pushing. With this we can then consider the case of motion of chains along their length. This topic was studied quite a lot in the 1850s due to the industrial interest in laying submarine cables for telecommunications at the time. By 1860, problems involving the motion of chains or ropes along their length made it into main-stream mechanics textbooks. In the case of a rope of mass per unit length λ and under tension T going around a pulley at a speed v, it can be shown that the speed and the tension are related in such a way that the pulley is not needed. In this case, it is found that the force that generates the centripetal acceleration to aid the motion of the chain or rope along the curve is the tension T and is equal to λv^2. The relationship between the tension and the speed of the chain being independent of the radius of the curved path taken implies that the chain can take any given path and the pulley can be done away with; in fact, any other given curved path the chain may take is stable.

We then consider a model for the chain with mass per unit length λ moving at a constant velocity v in a fountain in parts below:

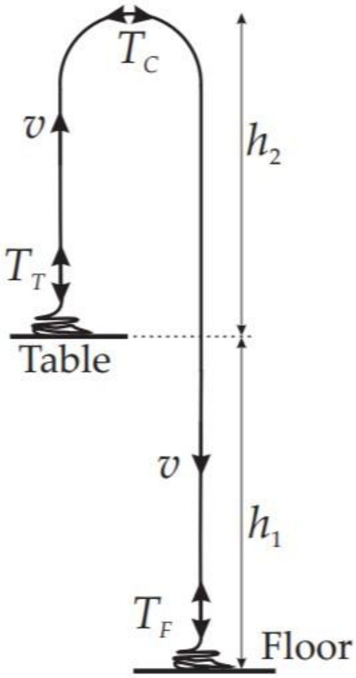


Figure 12. Simple model of chain fountain

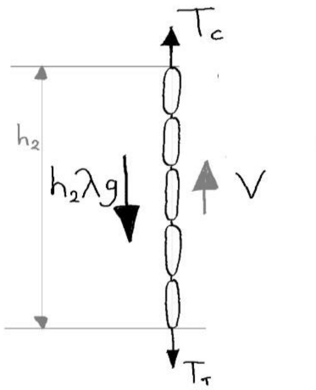
The apex of the fountain

At the apex of the fountain, as previously shown, the tension force is that which provides the centripetal force for the chain to move along the curved path. As such,

$$T_c = \lambda v^2 \qquad (1.1)$$

The rising part of the chain

The forces acting on the vertical portion of the chain above the container or pot have to balance since the chain is moving with a constant velocity.



Weight of chain in this portion is given by

$$h_2\lambda g$$

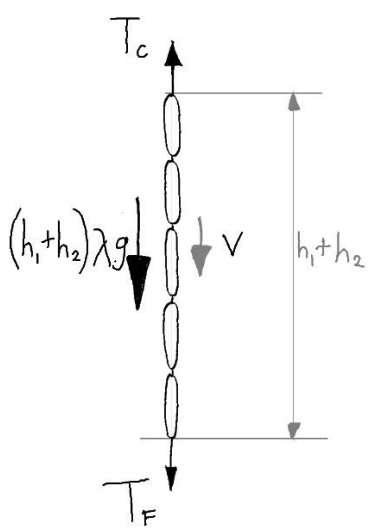
Balancing the forces gives

$$T_c = T_T + h_2\lambda g \qquad (1.2)$$

Figure 13. Forces on part of chain rising from container.

The falling part of the chain

Similarly the forces acting on the falling portion of the chain from the apex to the floor balance because of the constant velocity as shown in figure 3.



$$T_C = T_F + (h_1 + h_2)\lambda g \qquad (1.3)$$

Figure 14. Forces on part of chain falling from top of fountain.

The force required to accelerate the chain to a velocity v from the container on a table is the tension T_T which must be λv^2 as previously discussed. Subsequently, substituting this back into equation (1.2) which is the rising part of the chain, it is evident that there shall not be a fountain because $h_2 = 0$.

As such, propositions for the origin of the force that leads to this counterintuitive leap of the chain out of the container in which it is placed have been made. These propositions include a reaction force due to the nature of the rotation of a single chain link as it is just being pulled into motion looking at a much-simplified model of a chain developed by Dr. Biggins and Dr. Warner. Another proposition from Rogério Martins suggests that a kickback or backlash before a turn along the length of the chain during the process of unfurling as the chain flows out of the container, in addition to the reaction of the bottom of the container suggested by Dr. Biggins and Dr. Warner are the origin of the extra force required to cause the chain to leap into a fountain.

Biggins and Warner first predict that the extra force to enable the chain to leap into a fountain originates from the tension within the connections of the chain. They then model different arrangements of chain to further assist their understanding of the motion of the chain as it flows out of the container; they model a chain of metallic balls connected with long pieces of thread, as well as a chain of spaghetti. They finally simplify the chain to be one of long blocks connected by short pieces of thread. With this model, it was clear to see that a piece of chain (block in this model) just being pulled into motion would tend to move upwards while at the same time rotate about its center of mass. This rotation about the center of mass would mean that at some point, part of the chain would rotate below the floor of the container in which the chain rests. However, this cannot happen. Therefore, the container exerts a reaction force onto the piece of the block that is apparently rotating below the bottom of the container, as such pushing the piece of chain upwards, hence the fountain. Considering the simplified model yields the following.

Link of chain just being brought into motion

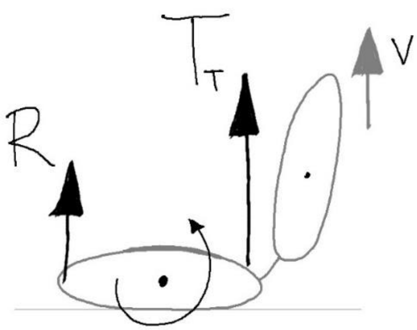


Figure 15. Chain link just being pulled into motion.

A link that is just being pulled into motion, is forced to rotate about its center of mass as it is being pulled by the tension. Since it cannot rotate into the pot, an upward reaction force ***R*** is introduced.

Thus $$T_T + R = \lambda v^2 \quad (1.4)$$

For dimensional consistency, ***R*** should be proportional to λv^2

$$R = \alpha \lambda v^2 \quad (1.5)$$

Link of chain coming to a stop

There has been observation of falling chains to accelerate faster than the acceleration due to gravity when falling onto a table or ground (Hamm E, Géminard J-C, 2010 and Grewal *et al*, 2011). When two equal lengths of hanging chain are set up in such a way that one falls into free space and the other onto a table when released from the same height, the chain falling into free

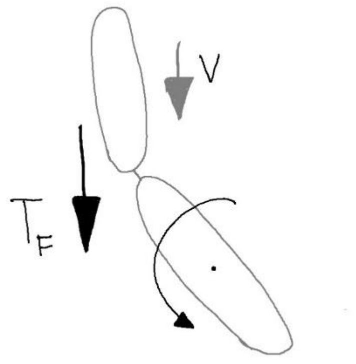


Figure 16. Chain link being brought to rest at the floor.

space accelerates with the acceleration due to gravity while the chain falling onto the table surprisingly accelerates faster than the acceleration due to gravity. The faster acceleration can only be accounted for and explained by an additional force that acts on the falling chain.

As such, a finite tension is introduced at the falling end of the chain and is also proportional to $\boldsymbol{\lambda v^2}$

$$T_F = \beta\lambda v^2 \qquad (1.6)$$

The obtained equations are solved to derive equations that predict the height of the fountain and the relationship between the initial height of the container in which the chain rests from the ground and the maximum height to which the chain would reach during its motion in a fountain.

$$\frac{h_1}{h_2} = \frac{\alpha}{(1-\alpha-\beta)} \qquad \text{and} \qquad v^2 = \frac{h_1 g}{(1-\alpha-\beta)}$$

In their analysis and experimentation, Biggins and Warner find the heights h_1 and h_2 to be related such that $h_2 = 0.14h_1$

Rogério Martins on the other hand suggests that the origin of this force is a kickback or backlash in the unfolding of the chain as it pours out of the container. Noticing that a well coiled up chain in a container does not leap as high as the randomly placed chain in the container, he proposes that the nature of the unfolding of the chain could be another factor for the force that causes the chain to leap over and above the rim of the container into a fountain. In the setup of arranging a chain horizontally on a table and pulling on one end of the chain to allow the chain to fall or flow to the ground and analyzing the motion of the chain frame by frame, Martins recognizes that there is a bigger kickback experienced along the length of the chain before a turn when multiple lines of the chain are arranged on the table compared to when simply a single line of chain is placed on the table. From observing the motion in this setup frame by frame, in addition to the observation that a well coiled up chain experiences a smaller height of the fountain, the conclusion is that the manner or order in which the chain is placed in the container is a contributing factor in the height to which the fountain reaches.

The objective of this thesis is to understand the propositions presented as well as study, analyze and simulate the motion of the chain so as to arrive at a better understanding of the mechanics involved in this exciting and entertaining fountain effect.

CHAPTER II: MATERIAL AND METHODS

In order to set up the experiment, a number 10 stainless steel ball chain was used. The ball chain is 100 feet long and each ball was 4.8mm or 3/16 inch in diameter. For the experiments, the chain was loaded into a plastic cup from which one end of the chain was given a slight push to set off the motion of the chain. The experiment was set up at the top of a staircase from a height of 3.6 meters. Another shorter number 3 nickel plated steel chain with diameter 3/32 inch or 2.4mm was used to compare observations. The same setup location was used for both chains.

Given that the origin of the force that causes the fountain is still somewhat unclear, different setups were used in an attempt to examine this still amazing chain fountain phenomenon. The basic setup is to place a pile of chain in a cup and either hold it in the air or place it on a raised surface such as a table. In the experiments, as explained, the setup was at the top of a staircase with the chain flowing to the very bottom. One setup was the basic chain in a plastic cup. The other setup employed a plastic cup but with some soft material placed at the bottom of the cup before loading the cup with the chain. The material used was polyester, which is the material sometimes used to stuff couch pillows. This provided a softer bottom surface for the chain rather than the firm plastic at the bottom of the cup. Another set up involved releasing the chain from a ceramic plate. Here, the smaller chain was used and the comparison between the fountain of a coiled-up chain and another arrangement where the chain was carefully placed onto the plate in circles was made. Several takes of setting the chain into motion were done and many of the observations were recorded in slow motion since the fountain happens rather quite fast even with the 100 feet long chain.

At the same time, a simple chain simulation was prepared to assist in the study and examination of the chain fountain. To run a simulation, a conducive programming language in which the simulation will be written has to be identified. The minimal installer for Anaconda, known as Miniconda which is the package that includes only conda, Python, the packages conda and python depend on, as well as a small number of other useful packages is the package that was used for the Python and Jupyter installations that were used. Anaconda is basically a distribution of Python and R programming languages for scientific computing which seeks to simplify package management and deployment. The simulation was written in Python language and run in Jupyter using the Miniconda command line, also known as Anaconda Prompt(miniconda3). Jupyter is an open-source web tool known as a computational notebook. The Jupyter notebook provides an interactive data science environment across many programming languages that does not only work as an Integrated Development Environment (IDE) but allows you to create and share documents that contain live code, equations, visualizations, and narrative text.

Using an Integrated Development and Learning Environment (IDLE) for Python, the simulation was written, and the program run in Jupyter through the Anaconda Prompt terminal and the motion was studied and examined to provide further assistance in the understanding of the chain fountain. The simulation took some time to be developed, however, versions of the proposed simulation were run as well as different configurations, such as a chain placed in a line horizontally on a table and allowed to flow to the ground.

For the simulation, the system-specific parameters and function (sys) module, pygame and Pymunk, as well as numpy and the random module were imported for use. The sys module provides access to some variables used or maintained by the interpreter and to functions that interact strongly with the interpreter. Pygame is a set of Python modules designed for writing video games; in our case we use it to aid with visualization. Pymunk on the other hand is a pythonic 2-dimension physics library that can be used whenever you need 2-dimension rigid body physics from Python. It works well when you need 2-dimensional physics, which is the nature of space we use. The random module generates and implements pseudo-random number generators for various distributions. We use this particular module to supply random velocities in the horizontal direction for the balls that make up the chain. Numpy is the fundamental package for scientific computing in Python. It is a Python library that provides a multidimensional array object, various derived objects, and an assortment of routines for fast operations on arrays, including mathematical, logical, shape manipulation, sorting, selecting, discrete Fourier transforms, basic linear algebra, basic statistical operations, random simulation and much more.

The chain is designed to have 120 balls piled up in rows of ten balls. The balls are constrained by pin joint to be 15 units apart as they execute their motion. The initial horizontal velocity of the balls is given a slight random amount to account for the sideways motion of the balls else the simulation would have them moving strictly in a vertical direction under gravity. The gravity in our simulation space or environment in which the simulation runs is set to 24,000 units with the positive axis pointing downwards, much like the usual treatment of the acceleration due to gravity as positive downwards. The balls are circles of radius 7 units and mass of 8.8 units. This setting of the radius was strategic in such a way as to be consistent with the number 10 chain. The stainless-steel ball chain has balls of diameter of 4.8mm with a distance between two consecutive balls being about 6mm. Therefore, setting out chain to have balls of 14 units in diameter placed 15 units apart is consistent with our stainless-steel ball chain. The elasticity applied to the balls, the container and ground was set to 1 since according to how the program manipulates the elasticity parameter, setting the elasticity to 1 comes closer to a more physically correct or acceptable simulation. Furthermore, the ball chain is designed to flow out of a container placed at a raised position from the ground in agreement with our observation of the chain flowing or falling from a higher altitude to a lower one.

For the horizontal chain, the parameters in the design code are set exactly the same as those for the piled chain. The only difference, however, is that the chain is not piled but rather laid in a long horizontal line on a horizontal surface that is also raised from the ground.

CHAPTER III: RESULTS AND DISCUSSIONS

The Mould effect is quite an amazing phenomenon, and any attempts in learning the not so simple dynamics of the fountain effect require much observation and analysis of the motion involved therein. We shall consider the different configurations as follows.

Chain in a cup on a table

Here, the chain fountain was clearly observed as expected. Given the approximated relationship between the fountain height and the height of the cup from the ground, which was 3.6m in our experiment, the chain reached a height of approximately 0.5m as shown below.

Figure 17. Chain fountain from cup.

Chain in cup with padding at the bottom

Similarly, yet shocking as well, a fountain of up to approximately 0.5m was observed when some soft material was placed at the bottom of the cup before the chain was loaded. The result is shown below.

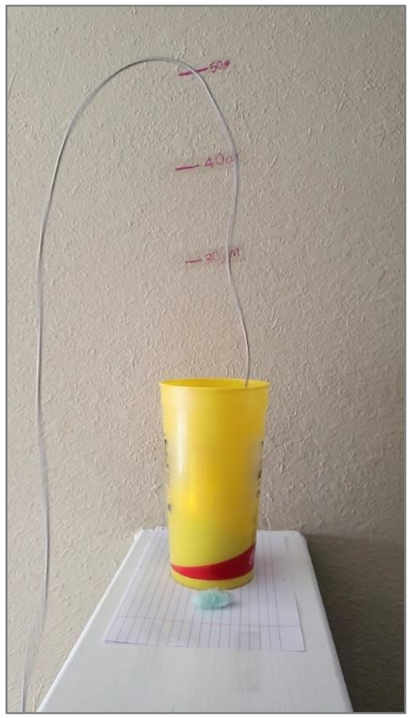


Figure 18. Chain fountain from cup with padding.

Chain arranged in a neat circular coil and neat horizontal lines on a plate

Here, the smaller chain was used. The fountains observed when the chain was arranged in a neat circular coil and neat horizontal lines was approximately equal as shown below.

Figure 19. Showing the neat coil and horizontal line arrangement, as well as the chain fountain from the horizontal lines arrangement.

Chain randomly placed on a plate

In this arrangement, the smaller chain was again used for easier comparison with the previous arrangements. The fountain observed here was significantly higher than the previous arrangements as shown below.

Figure 20. Fountain from randomly arranged chain.

Simulation with horizontal chain

Although fundamentally different from the chain fountain in that the chain in this case is not piled up, the chain was seen to accelerate – as expected – first around the edge of the raised surface and eventually started flowing around a path some distance away from the edge as shown in the frames below.

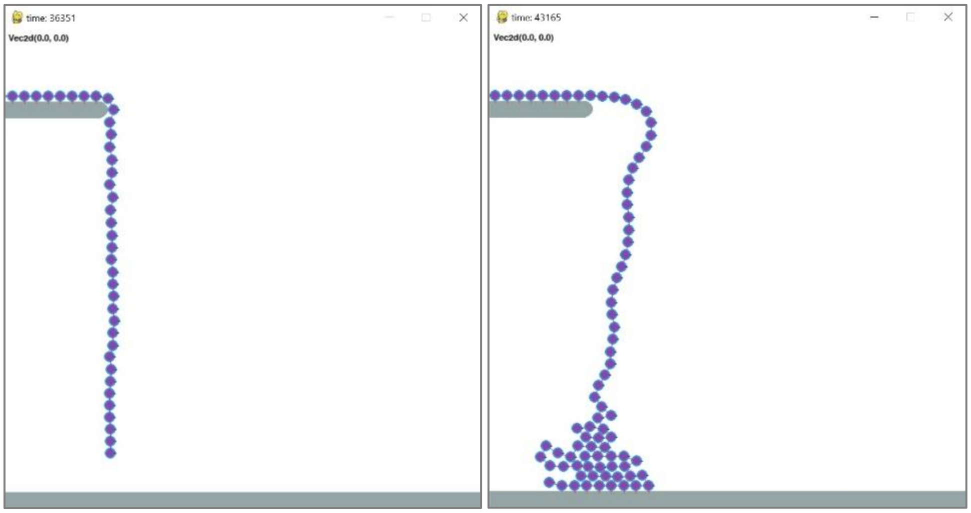


Figure 21. Frames showing the simulation of freely falling chain placed horizontally on a table.

Simulation with piled chain

With the careful specifications put into the design of the chain fountain simulation, the observations made were fairly consistent with the physically observed results. Different specifications such as the mass of the balls that make up the chain, the dimensions of the chain, even the elasticity of the chain, value for gravity and difference in height from the ground to the pot were altered and reassuringly, a slight chain fountain was observed with a larger container height.

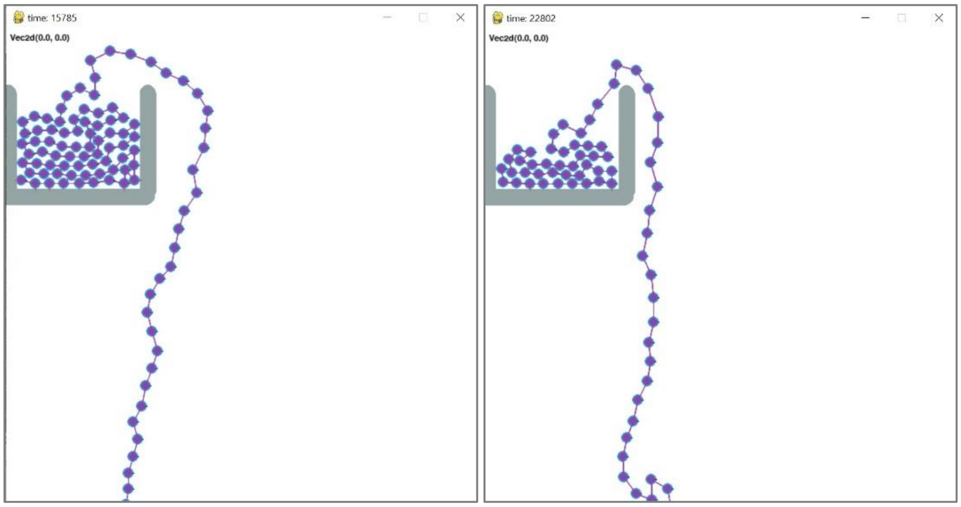


Figure 22. Frames showing the fountain by the chain simulation.

The chain fountain remains a marvel to watch, and it gets more interesting considering the number of possible paths the chain could take including those observed as the chain is unfurling from the pile during pickup as the already falling portion of the chain siphons the remaining portion from the pile. From the experiments, it was observed that the fountain starts off at a lower height and gradually – sometimes more rapidly – increases in height as the chain accelerates until it reaches a relatively constant velocity at which the maximum fountain height is usually attained. The last portion of the chain is observed to leave the cup in what appears to be like a whip which is similar to the kick in the tip of the free end at the end of the falling of a chain when one end is released from a height above the other end that is fixed at a point being released from an initial shape such that there is some portion of the chain arching beneath the fixed point. This kick as the last links of the chain are pulled out of the cup clearly illustrates, if you may, the additional reaction force from the cup or table as illustrated by Biggins and Warner. This coupled with the fact that the chain has a maximum local curvature, which property was carefully incorporated in the simulation, allowing for the simplification of the bead chain to one made up of rods with one rod, for example, covering the length of 3 beads. With such a simplification, the reaction force from the cup or table is easier imagined and explained. Why the chain with the padding at the bottom of the cup is able to trace a fountain in its motion is also easily explained because the beads at the top of the pile are resting on the beads beneath them which then become the source of this reaction force before they are pulled into motion and the process continues until the whole chain is pulled out of the cup. Furthermore, the property of the chain having a maximum local curvature with the simplification of the chain as one made of rods explains why the arrangement of the chain plays a part in the final observed maximum fountain height. With the chain arranged neatly, for example in a coil or neat horizontal lines, the unfurling process is smoothened, whereas with a random arrangement, there are a series of kicks as the chain is being straightened while being pulled into motion and these kicks add significantly to the extra reaction to push the chain into a fountain as observed in the significant difference between the fountain heights of the neatly arranged chain and one that is randomly arranged. The nature of arrangements also explains why the simulation has only a slight fountain because the chain was designed to be loaded in the form of lines of 10 beads layered on top of each other.

Our simulation experiment therefore agrees with the physically observed proposed sources of the extra force required to push the chain into a fountain. The simulation is really interesting because although it does not account for the complexity of the problem at hand; the dynamics of the motion in question, which dynamics are indeed sophisticated because the motion of the chain tracing the path of a fountain as observed requires the consideration of not only the conservation of momentum but also constrained motion. Numerical simulations that have been done for problems such as this one offer more correct predictions and results pointing to the formation of the chain fountain. It remains evident that the motion of the chain is set off by the release of gravitational potential energy and this explains the gradual increase of the chain's velocity solely because of the acceleration due to gravity.

CHAPTER IV: CONCLUSIONS

After much experimentation and several simulation runs, it is made clear that the motion of a chain in a fountain when one end is released from a container raised a distance above the ground is caused by an anomalous reaction experienced by the portion of the chain that is just being pulled into motion. This reaction force is also dependent on the arrangement of the chain while being loaded or placed into the container because it is sufficiently dependent on the property of the chain having a local maximum curvature. This is supported by the fact that there is no observed fountain for a chain that has no local maximum curvature such as one that is made of balls connected by long strings.

Special treatment had to be accorded to the design of the chain fountain simulation in order to arrive at results that are as consistent as possible with what is physically observed. As such, not only was the choice of program and parameters key to the design, but also learning how these different parameters were intricately related as well as the program's treatment of the various parameters was essential and critical in the design. Although a more complex approach affording attention to the subtle details of the motion involved in the motion of the chain fountain, such as the acceleration of the chain, as well as the much more sophisticated chain dynamics, will give a more detailed and conclusive result, the simulation developed in this experiment is sufficient to provide the results that we can work with to fulfil the objective of learning the motion of a chain in what is known as the Mould effect in a steady state.

The process for the development and design of the simulation was indeed valuable as it helped with the understanding of how complex the problem is, and this was important in the efforts of setting the parameters in different ways and understanding further what contribution different properties of the chain have in the motion of the chain fountain. For example, to alter the tension in the chain, the mass of the balls is the parameter that is altered since the tension is dependent on the mass per unit length as discussed earlier. The design of the simulation itself is something to appreciate because of the lessons involved in the process such as learning a programming language, the handling of different packages and modules and how all these interact with each other resulting in the end that a visual simulation is delivered for viewing which assists us in the study of the motion at hand. It is also important to appreciate how much the calculation the program does in handling such a number of bodies as we have in the simulation while respecting the condition of constrained motion. For more accurate resultant simulation, a much smaller time step, which is simply an incremental change of time for which the program performs the required calculations observing motion and change in momentum with constraints, has to be used. However, the smaller the time step, the much longer the program takes in rendering the simulation. The choice of time step used in this experiment is sufficient for analysis to be carried out. Thus, the consideration of this fountain effect in a simulation provides not only the opportunity to learn more about the motion in question, but also the necessary aspects required in developing such simulations.

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